13장 만유인력

4. $m_1 + m_2 = 5.00\ \text{kg}$이므로, $m_2 = 5.00\ \text{kg} - m_1$ 이용

$$F = G\frac{m_1 m_2}{r^2} = 1.00 \times 10^{-8}\ \text{N}$$

$$1.00 \times 10^{-8}\ \text{N} = (6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)\frac{\text{m}_1(5.00\ \text{kg} - \text{m}_1)}{(0.200\ \text{m})^2}$$

$$(5.00\ \text{kg})\text{m}_1 - \text{m}_1^2 = \frac{(1.00 \times 10^{-8}\text{N})(0.0400\ \text{m}^2)}{6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2} = 6.00\ \text{kg}$$

$$m_1^2 - (5.00\ kg)m_1 + 6.00\ kg = 0,\ (m_1 - 3.00\ \text{kg})(\text{m}_1 - 2.00\ \text{kg}) = 0$$

$m_1 = 3.00\ \text{kg}, \text{m}_2 = 2.00\ \text{kg}$ 또는 $m_1 = 2.00\ \text{kg}, \text{m}_2 = 3.00\ \text{kg}$

6. $g_e = G\frac{M_e}{R_e^2}$, $g_m = G\frac{M_m}{R_m^2}$, $g_m = \frac{1}{6}g_e$, $R_m = 0.250\,R_e$

$$\frac{M_m}{(0.250\,R_e)^2} = \frac{1}{6}\frac{M_e}{R_e^2} \qquad \therefore M_m = \frac{1}{6}(0.250)^2 M_e$$

$$\frac{\rho_m}{\rho_e} = \frac{M_m/R_m^3}{M_e/R_e^3} = \frac{\frac{1}{6}(0.250)^2 M_e/(0.250R_e)^3}{M_e/R_e^3} = \frac{(0.250)^2}{6(0.250)^3} = \frac{1}{6(0.250)} = \frac{2.00}{3} = 0.667$$

9. $F_g = mg = G\frac{M_E m}{r^2}$ 이용,

$g = G\frac{M_E}{r^2}$에서 중력 가속도가 $9.00\ \text{m/s}^2$ 이므로,

$$r = \sqrt{\frac{GM_E}{g}} = \sqrt{\frac{(6.67 \times 10^{-11}\ N \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{9.00\ \text{m/s}^2}} = 6.66 \times 10^6\ \text{m}$$

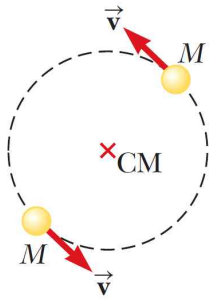
케플러의 제 3법칙을 이용하여,

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{(6.66 \times 10^6\ \text{m})^3}{(6.67 \times 10^{-11}\ N \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}}$$

$$= 5.41 \times 10^3\ \text{s}$$

$$T = (5.41 \times 10^3\ \text{s})\left(\frac{1\text{h}}{3600\ \text{s}}\right) = 1.50\,\text{h} = 90.0\ \text{min}$$

11.



별간의 만유인력이 구심력이 되므로

$$f = M\frac{v^2}{r} = G\frac{M^2}{(2r)^2} \quad \cdots ①$$

공전주기 $T = 14.4 \times 24 \times (60)^2$ s이고,

$vT = 2\pi r$ 이므로, $r = \dfrac{vT}{2\pi}$ $\cdots ②$

①과 ②로부터, $M = \dfrac{4rv^2}{G} = \dfrac{2v^3T}{\pi G} = 1.26 \times 10^{32}\,\text{kg}$

17. 초기속력 $v_i = 2.00 \times 10^4\,\text{m/s}$, 만유인력 상수 $G = 6.674 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$

지구질량 $M_e = 5.97 \times 10^{24}\,\text{kg}$, 지구반지름 $R_e = 6.37 \times 10^6\,\text{m}$

지구-로켓 계에 대해 $E_i = E_f$로부터,

$$\frac{1}{2}mv_i^2 - \frac{GM_em}{R_e} = \frac{1}{2}mv_f^2 \qquad \therefore v_f = \sqrt{v_i^2 - \frac{2GM_e}{R_e}} = 1.66 \times 10^4\,\text{m/s}$$