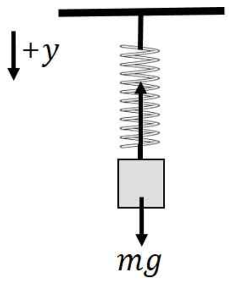
15장 진동

2. $m = 0.010\ \text{kg}$인 물체가 용수철에 매달려 $l = 3.9\ \text{cm}$만큼 늘어난 경우, 물체에 미치는 연직방향(y)으로의 알짜힘은 영이므로,



$$\sum F_g = mg - kx = 0 \qquad \therefore \text{용수철의 힘상수 } k\text{는 } k = \frac{mg}{l}$$

$m' = 0.025\ \text{kg}$인 물체를 바꾸어 달고 연직방향으로 단조화 운동을 시킬 때의 주기를 T라고 하면,

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m'}{k}} = 2\pi\sqrt{\frac{m'l}{mg}} = 0.63\ \text{s}$$

8. $k = 8.00\ \text{N/m},\ m = 0.500\ \text{kg},\ A = 0.100\ \text{m} \ \rightarrow \omega = \sqrt{\frac{k}{m}}$

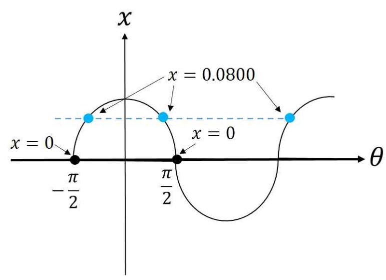
(a) $v_{\max} = \omega A = A\sqrt{\frac{k}{m}} = 0.400\ \text{m/s}$

(b) $a_{\max} = \omega^2 A = A\frac{k}{m} = 1.60\ \text{m/s}^2$

(c) $x(t) = A\cos(\omega t + \phi) = 0.0600\ \text{m} \ \rightarrow \omega t + \phi = \cos^{-1}\left(\frac{0.0600}{\text{A}}\right)$

$v(t) = -A\omega\sin(\omega t + \phi)$ 이므로, $|v(t)| = A\omega\sin\left(\cos^{-1}\left(\frac{0.0600}{A}\right)\right) = 0.320\ \text{m/s}$

(d) $a(t) = -A\omega^2\cos\left(\cos^{-1}\left(\frac{0.0600}{A}\right)\right) = -0.960\ \text{m/s}^2$



(e) ① $x_i = 0 = A\cos(\omega t_i + \phi) \rightarrow \omega t_i + \phi = \theta_i = \pm\frac{\pi}{2} + 2n\pi$, n은 정수

② $x_f = 0.0800 = A\cos(\omega t_f + \phi) \rightarrow \omega t_f + \phi = \theta_f = \pm\cos^{-1}\left(\frac{0.0800}{A}\right) + 2n\pi$

③ x가 $0 \rightarrow 0.08$로 변할 때 $\Delta\theta$를 최소로 하기 위한 조건은 $\theta_i = -\frac{\pi}{2} + 2n\pi,\ -\frac{\pi}{2} + 2n\pi < \theta_f < 2n\pi$

이므로, $\theta_f = -\cos^{-1}\left(\frac{0.0800}{A}\right) + 2n\pi$

$$\therefore \theta_f - \theta_i = -\cos^{-1}\left(\frac{0.0800}{A}\right) + \frac{\pi}{2} = (\omega t_f + \phi) - (\omega t_i + \phi) = \omega(t_f - t_i)$$

$$t' - t = \frac{1}{\omega}\left[-\cos^{-1}\left(\frac{0.0800}{A}\right) + \frac{\pi}{2}\right] = 0.232\ \text{s}$$

11. 진폭 2배: $A' = 2A$,

(a) $E = \frac{1}{2}kA^2 \rightarrow E' = \frac{1}{2}k(2A)^2 = 4E$

(b) $v_{\max} = \sqrt{\frac{k}{m}}A \rightarrow v'_{\max} = 2v_{\max}$

(c) $a_{\max} = \frac{k}{m}A \rightarrow a'_{\max} = 2a_{\max}$

(d) $T = 2\pi\sqrt{\frac{m}{k}}$ → 주기에는 영향을 끼지치 않는다.

12(a) $E = \frac{1}{2}kA^2 = 2.80\times10^{-2}\ \mathrm{J}$

(b) $x(t) = A\cos(\omega t+\phi) = 0.0100\ \mathrm{m}\quad \therefore \omega t+\phi = \pm\cos^{-1}\left(\frac{0.0100}{A}\right)$

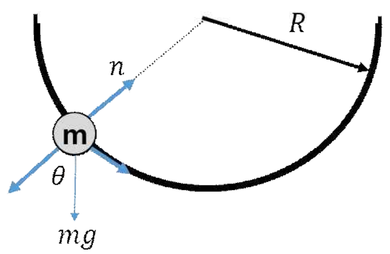
$v(t) = -A\omega\sin(\omega t+\phi) = -A\omega\sin\left(\pm\cos^{-1}\frac{0.0100}{A}\right) = \mp 1.02\ \mathrm{m/s}\ ,\ |v(t)| = 1.02\ \mathrm{m/s}$

(c) $x(t) = A\cos(\omega t+\phi) = 0.0300\ \mathrm{m},\ \therefore \omega t+\phi = \pm\cos^{-1}\left(\frac{0.0300}{A}\right)$

$v = \mp A\omega\sin\left(\cos^{-1}\left(\frac{0.0300}{A}\right)\right),\ K = \frac{1}{2}mv^2 = 1.23\times10^{-2}\,\mathrm{J}$

(d) $x = 0.0300\ \mathrm{m},\ U = \frac{1}{2}kx^2 = 1.58\times10^{-2}\ \mathrm{J}$

16.



$\sum F_c = n - mg\cos = 0$

$\sum F_t = -mg\sin\theta = ma$

$\frac{d^2s}{dt^2} = -g\sin\theta\quad S = R\theta$이므로, $\frac{d^2\theta}{dt^2} = -\frac{g}{R}\sin\theta$

작은 변위에서 $\rightarrow \theta \ll 1 \rightarrow \sin\theta \approx \theta$

$\therefore \frac{d^2\theta}{dt^2} = -\frac{g}{R}\theta = -\omega^2\theta \qquad \omega = \sqrt{\frac{g}{R}}$